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A NOTE ON DAMAGE-BASED INELASTIC SPECTRA

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SUMMARY

When a structure is subjected to moderate to severe ground motions, a few excursions of the response yield level may take place following the reductions usually enforced in the design forces. These excursions are associated with progressive damage in the structure. Thus, a choice of the design level has to be suitably based on the maximum damage to be allowed in the structure. In this paper, a stochastic technique of developing damage-based non-linear spectra has been proposed for the aseismic design of those structures which can be idealized through Single-Degree-Of-Freedom (SDOF) oscillators. The proposed technique has been illustrated by obtaining the non-linear spectra which can possibly be used for a damage-based design. Along with the spectra, allowable ductility demand which should be supplied through proper sizing and detailing of the members and is compatible with the damage has also been specified. The non-linear SDOF oscillators have been approximated for this purpose by equivalent linear oscillators using a new stochastic linearization technique. The proposed linearization technique has been validated through simulation results in the case of an idealized, non-hysteretic, Elasto-Plastic (EP) model.

KEY WORDS: inelastic excursions; equivalent linearization; peak statistics; inelastic spectra; cumulative damage; ductility

INTRODUCTION

A popular approach for the aseismic design of structures is to use the elastic response spectra and to reduce the design forces for economical reasons, thus permitting a few inelastic excursions during severe earthquakes. These reductions primarily depend on the provided or tolerable ductility and on the overstrength available in the structure. The reduced design forces in the case of a Single-Degree-Of-Freedom (SDOF) oscillator are represented in the form of inelastic strength spectra for a given ductility demand. Further, for controlling the inelastic performance of the structure, it is also essential that the maximum relative displacement is limited within specified bounds. Thus, inelastic displacement spectra have also been introduced for the structural design. However, when a system is subjected to different numbers of non-linear excursions, it may undergo different degrees of damage, even though the ductility demand or the maximum inelastic displacement remains the same. Thus, it is essential for the present aseismic design practice to incorporate a measure of the cumulative damage in the inelastic spectra such that these spectra not only provide information about the design forces and maximum inelastic displacements but also about the associated damage.

Vidic *et al.*[1] proposed strength and displacement compatible spectra by carrying out parametric studies for several ground motions. Fajfar and Vidic[2] incorporated the parameters of input and hysteretic energy in estimating the inelastic spectra for strength and displacement. Fajfar[3] proposed equivalent ductility factors based on damage due to low-cycle fatigue and used those for constructing the inelastic spectra. Mahin and Bertero[4] carried out time-history analyses to obtain inelastic spectra and studied the effects of various governing factors on them. Their study has shown that for a set of non-linear spectra, the ductility demand and the number of yield events (equal to the number of inelastic excursions) are quite sensitive to the parametric variations as well as to the type of non-linear oscillator under consideration. Mostaghel and

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Hernried[5] proposed bounds on the inelastic spectral values but their results were conservative for some ranges of natural period of an elasto-plastic oscillator. Miranda[6,7] studied the effects of site and soil conditions on the inelastic spectra. Murakami and Penzien[8] presented constant strength probabilistic inelastic spectra based on artificially generated accelerograms. Iwan[9] proposed relations to obtain the inelastic spectra from the elastic spectra by using the equivalent linearization theory. Park *et al.*[10] proposed a damage index based on plastic displacement and energy dissipation capacity and used it for the limiting design of reinforced concrete structures. This model, popularly known as the Park–Ang model, is widely used as it is calibrated and is also simple. Cosenza *et al.*[11] used different damage models to obtain collapse spectra (i.e. the spectra corresponding to the complete collapse) and carried out a comparison between the corresponding damage functionals. Several other researchers, e.g. McCabe and Hall,[12] Uang and Bertero[13] and Nassar *et al.*,[14] have also contributed significantly to the concept of damage-based inelastic spectra. None of these studies has however explicitly considered the cumulative damage occurring from the repeated inelastic excursions. As shown by Basu and Gupta,[15] these excursions may directly influence the progressive damage of structures.

In the present paper, a stochastic approach has been proposed to compute the damage-dependent inelastic spectra. These spectra may be useful in the design of structural systems with better balance and control, keeping in view several practical factors such as the frequency of earthquakes, importance of the structure and the economical losses associated with the structural failure. The proposed approach is based on the formulations of Basu and Gupta[15,16] on the cumulative progressive damage and conditional ductility. The non-linear oscillators have been linearized for this purpose by using a new stochastic linearization technique. The formulation of this new technique is based on the equivalence of the maximum response instead of the response in the mean square sense. The results from this technique are validated by digital simulation using the synthetically generated and recorded accelerograms of four different earthquakes. The paper also includes the computations of the maximum allowable ductility demand for a specified total damage.

EQUIVALENT LINEAR OSCILLATOR

The formulation of the inelastic spectra presented in the paper is in terms of the characteristics of an equivalent linear oscillator. For this, the equivalent linearization technique is used wherein the choice of the parameters of the equivalent linear system is based on a specific criterion of equivalence of the linear oscillator with the non-linear oscillator. In the traditional linearization method, the condition imposed to obtain the equivalent system parameters is the equivalence of the mean square response, i.e. the system parameters are obtained by minimizing the mean square error between the linear and non-linear system equation. For this, the response probability distribution is assumed to be Gaussian. For a mildly non-linear oscillator, this assumption is reasonable and the statistics of the response of the equivalent oscillator may be assumed to be close to those of the non-linear oscillator. Iwan and Gates[17] and Iwan[9] have already used this technique for a class of oscillators to obtain the inelastic response.

Since the inelastic spectra provide the design forces via the maximum responses, the above linearization method is proposed to be modified, with the equivalence criterion now based on minimizing the mean square error of the largest maximum of the response process. It is to be noted that this would not guarantee the equivalence of the higher-order peaks, and, in fact, a more comprehensive linearization scheme should include the other factors which affect the cumulative damage. However, the maximum response of an oscillator is of prime interest to us, and this more relaxed criterion would lead to a better matching of the largest maximum of the equivalent linear response with that of the non-linear response.

Let us consider a non-linear oscillator characterized by a non-linear stiffness, $g(x)$, damping ratio, ζ, and initial natural frequency, ω_n. The equation of motion of this oscillator may be written as

$$\ddot{x} + 2\zeta\omega_n\dot{x} + g(x) = -\ddot{u}_g(t) \tag{1}$$

where $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$ are the displacement, velocity and acceleration, respectively, of the non-linear oscillator subjected to ground acceleration, $\ddot{u}_g(t)$. Let us consider an idealized Elasto-Plastic (EP) oscillator

having the form of $g(x)$ as

$$\begin{aligned} g(x) &= \omega_n^2 x_y, \quad x \geqslant x_y \\ &= \omega_n^2 x, \quad -x_y < x < x_y \\ &= -\omega_n^2 x_y, \quad x \leqslant -x_y \end{aligned} \tag{2}$$

where x_y is the yield displacement of the oscillator.

Let the equation of motion of the equivalent linear oscillator be given by

$$\ddot{x} + 2\zeta_e \omega_e \dot{x} + \omega_e^2 x = -\ddot{u}_g(t) \tag{3}$$

where ω_e is the equivalent linear natural frequency. Since the non-linearity is assumed to be in stiffness alone, the damping force does not change for the equivalent oscillator. Thus, corresponding to a change in the natural frequency, the damping ratio, ζ_e, has to be changed in the equivalent system so that the damping force remains unchanged. This can then be expressed as

$$\zeta_e = \frac{\omega_n}{\omega_e}\zeta \tag{4}$$

By taking the difference between equation (1) and equation (3) and squaring, the square of the error can be written as

$$\varepsilon^2 = (\omega_e^2 x - g(x))^2 \tag{5}$$

In the case of the traditional linearization, the expression for the expected value of ε^2 is minimized to obtain the parameters of the linear system. The expectation operation is performed with respect to the response probability density function which is usually assumed to be Gaussian as

$$\phi(x) = \frac{1}{\sqrt{2\pi}\, x_{rms}} e^{-(1/2)(x/x_{rms})^2} \tag{6}$$

where x_{rms} is the r.m.s. value of the response process, $X(t)$. Thus, the equivalent natural frequency can be obtained from

$$\frac{d}{d\omega_e^2}\int_{-\infty}^{\infty} (\omega_e^2 x - g(x))^2 \phi(x)\, dx = 0 \tag{7}$$

Simplifying the above expression and substituting the expression for $g(x)$ from equation (2), we can obtain the following relation:

$$\left(\frac{\omega_e}{\omega_n}\right)^2 = \frac{2\int_0^{x_y} x^2 \phi(x)\, dx}{x_{rms}^2} \tag{8}$$

This equation has to be solved iteratively by assuming an initial value of x_{rms}, e.g. same as that of the linear response.

In the case of the proposed linearization technique, the square of the error, ε^2, is sought to be minimized with the equivalence based on the largest maximum response. The probability density function (p.d.f.) describing the distribution of maxima of a random, zero mean, stationary, Gaussian response, $X(t)$, can be written as (see References 18–20)

$$p(\eta) = \frac{1}{\sqrt{2\pi}}\left[\varepsilon e^{-\eta^2/2\varepsilon^2} + (1-\varepsilon^2)^{1/2}\eta e^{-\eta^2/2}\int_{-\infty}^{\eta(1-\varepsilon^2)^{1/2}/\varepsilon} e^{-x^2/2}\, dx\right] \tag{9}$$

where η is the normalized peak value w.r.t. x_{rms}, such that

$$x_{rms} = \lambda_0^{1/2} \tag{10}$$

and

$$\varepsilon = \left[\frac{\lambda_0\lambda_4 - \lambda_2^2}{\lambda_0\lambda_4}\right]^{1/2} \tag{11}$$

is a measure of the bandwidth of the energy spectrum. λ_n is, in general, the nth moment of the energy spectrum, $E(\omega)$ of $X(t)$, and is defined by

$$\lambda_n = \int_0^\infty \omega^n E(\omega)\,\mathrm{d}\omega, \quad n = 0, 1, 2, \ldots \tag{12}$$

Using the formulation of Gupta and Trifunac,[21] the probability density function of the largest maximum becomes

$$q(\eta) = N[1 - P(\eta)]^{N-1} p(\eta) \tag{13}$$

where the total number of maxima, N, in the process duration, T, is given in terms of the moments of $E(\omega)$ as

$$N = \frac{T}{2\pi}\left[\frac{\lambda_4}{\lambda_2}\right]^{1/2} \tag{14}$$

and $P(\eta)$ is the cumulative probability given by $P(\eta) = \int_\eta^\infty p(u)\,\mathrm{d}u$. Thus, the proposed linearization scheme can be formulated as

$$\frac{\mathrm{d}}{\mathrm{d}\omega_e^2}\int_{-\infty}^{\infty} (\omega_e^2 x - g(x))^2 q(x)\,\mathrm{d}x = 0 \tag{15}$$

On simplification of the above equation and substitution of $g(x)$ from equation (2), we obtain

$$\left(\frac{\omega_e}{\omega_n}\right)^2 = \frac{x_y\left[\int_{x_y}^{\infty} xq(x)\,\mathrm{d}x - \int_{-\infty}^{-x_y} xq(x)\,\mathrm{d}x\right] + \int_{-x_y}^{x_y} x^2 q(x)\,\mathrm{d}x}{\int_{-\infty}^{\infty} x^2 q(x)\,\mathrm{d}x} \tag{16}$$

On normalizing the response with respect to the r.m.s. value, x_{rms}, the above equation may be written as

$$\left(\frac{T_n}{T_e}\right)^2 = \frac{\eta_y\left[\int_{\eta_y}^{\infty} \eta q(\eta)\,\mathrm{d}\eta - \int_{-\infty}^{-\eta_y} \eta q(\eta)\,\mathrm{d}\eta\right] + \int_{-\eta_y}^{\eta_y} \eta^2 q(\eta)\,\mathrm{d}\eta}{\int_{-\infty}^{\infty} \eta^2 q(\eta)\,\mathrm{d}\eta} \tag{17}$$

where

$$\eta_y = \frac{x_y}{x_{\mathrm{rms}}} \tag{18}$$

is the normalized yield displacement, T_e $(= 2\pi/\omega_e)$ is the period of the equivalent oscillator, and T_n $(= 2\pi/\omega_n)$ is the initial period of the EP oscillator. Equation (17) has to be solved iteratively with the initial guess values of ε, x_{rms} and N for a given yield displacement, x_y. Thus, in the proposed formulation, all parameters which affect the largest maximum of a process have been taken into account as opposed to x_{rms} only in the traditional formulation. It may also be noted that in both equations (8) and (17), ω_e and T_e, respectively, approach ω_n and T_n, as x_y tends to ∞.

Although the oscillator as considered in the above formulation does not truly represent the force–deformation relationships of most structures, this has been chosen arbitrarily to illustrate the new linearization scheme in a simple manner. Minor modifications would be necessary to make this applicable to more realistic situations with the structures having hysteretic energy dissipation capacities.

VALIDATION OF EQUIVALENT LINEARIZATION THROUGH DIGITAL SIMULATION

The proposed linearization technique has been validated by using the simulation results and has also been compared with the traditional linearization technique results. For the purpose of statistical simulation, 20

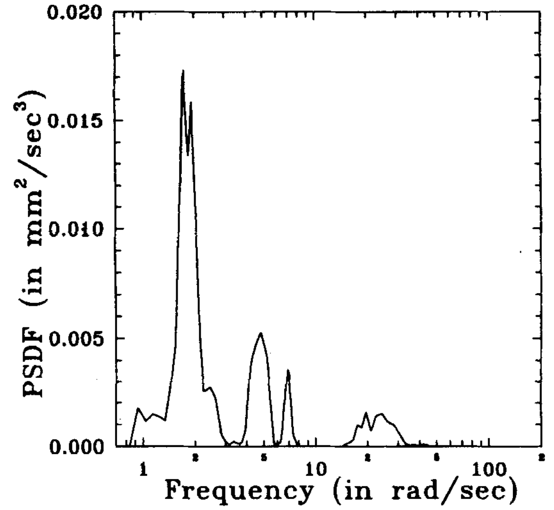

Figure 1. Power spectral density function for the motion at Dumbarton Bridge site during the Loma Prieta earthquake

synthetic accelerograms have been generated corresponding to the 1989 Loma Prieta earthquake for the Dumbarton Bridge site (near Coyote Hills) by using the SYNACC program by Wong and Trifunac[22] (see Reference 23 for details). These accelerograms are based on calculations of the group-arrival times of the waves of different frequencies and thus incorporate realistic representation of the non-stationary amplitude and frequency composition. Damping of the non-linear system has been assumed to be 5 per cent of the critical. Numerical simulation has been carried out using the fourth-order Runge–Kutta method with a time step of 0·02 sec, and the inelastic response spectra have been computed with the yield level of the EP oscillator arbitrarily taken as 250 mm. To characterize the ground motion through its PSDF, the mean (elastic) response spectrum has been computed using the ensemble of 20 accelerograms, and then the iterative scheme of Kaul[24] and Unruh and Kana[25] has been used for obtaining the response spectrum compatible PSDF of the ground motion. It may be noted that this spectrum-compatible PSDF, as shown in Figure 1, takes into account the non-stationarity in response both due to the inherent non-stationarity in the ground motion as well as due to the transient nature of the response. Using this, the frequency domain stochastic analyses are carried out to obtain the maximum responses for both the traditional and the proposed linearized oscillators. If $S_i(\omega)$ is the input PSDF to an equivalent oscillator with ζ_e and ω_e as the equivalent damping ratio and natural frequency, respectively, then the r.m.s. value of the response, $X(t)$, is given by

$$x_{\rm rms} = \left[\int_0^\infty \frac{S_i(\omega)}{(\omega_e^2 - \omega^2)^2 + (2\zeta_e\omega_e\omega)^2} \, d\omega \right]^{1/2} \tag{19}$$

By multiplying this with a suitable peak factor for the process, $|X|$, we can obtain the expected absolute maximum value of the response. The peak factor for the expected largest maximum is given by (see Reference 26 for details)

$$\eta^* = \int_0^\infty \eta \tilde{q}(\eta)\, d\eta \tag{20}$$

where

$$\tilde{q}(\eta) = \tilde{N}[1 - \tilde{P}(\eta)]^{\tilde{N}-1} \tilde{p}(\eta) \tag{21}$$

is the probability density function for the largest maximum in $|X|$. In this expression,

$$\tilde{p}(\eta) = \frac{1}{\sqrt{2\pi}} \left[2\varepsilon e^{-\eta^2/2\varepsilon^2} + (1-\varepsilon^2)^{1/2} \eta e^{-\eta^2/2} \int_{-\eta(1-\varepsilon^2)^{1/2}/\varepsilon}^{\eta(1-\varepsilon^2)^{1/2}/\varepsilon} e^{-x^2/2} \, dx \right], \quad \eta \geqslant 0 \tag{22}$$

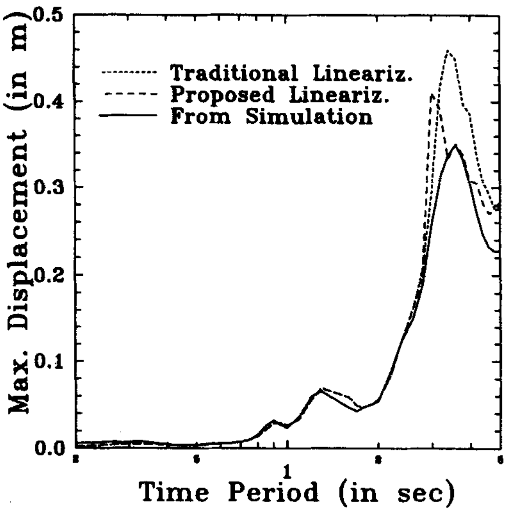

Figure 2. Comparison of expected maximum displacements from simulation and linearization techniques

is the probability density function for the maxima in the process, $|X|$, $\tilde{P}(\eta) = \int_{\eta}^{\infty} \tilde{p}(u)\,\mathrm{d}u$ is the cumulative probability, and

$$\tilde{N} = \frac{T}{2\pi}(1 + \sqrt{1 - \varepsilon^2})\left[\frac{\lambda_4}{\lambda_2}\right]^{1/2} \tag{23}$$

is the total number of expected peaks in $|X|$.

The expected peak displacements for the traditional and the proposed linear oscillators with varying values of T_n are compared with the results of the exact time-history analyses in Figure 2. It is seen that the estimates from both the linearization schemes are in nice agreement with those from the simulation, particularly in the short to medium period range. However, in the period range of 2–5 sec, marginal better agreement of the proposed linearization technique with the simulated inelastic response is observed through the better matching of the peaks and troughs. Since the proposed linearization scheme does not involve

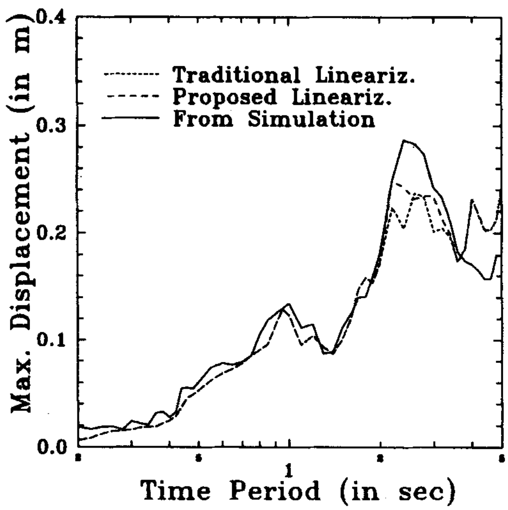

Figure 3. Comparison of expected maximum displacements for the Imperial Valley earthquake case

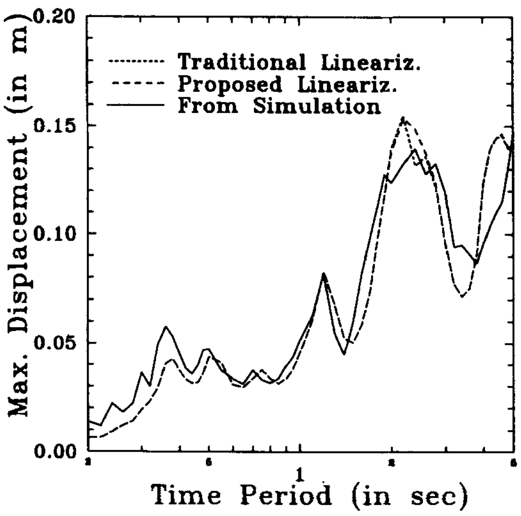

Figure 4. Comparison of expected maximum displacements for the Parkfield earthquake case

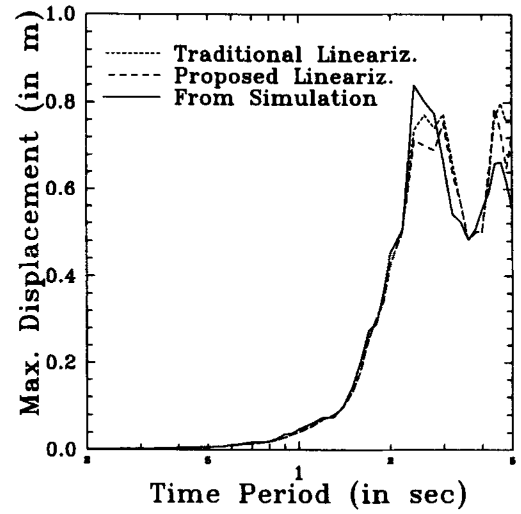

Figure 5. Comparison of expected maximum displacements for the Michoacan earthquake case

additional complexity and computational effort, this can be used in estimating the largest response as an alternative to the traditional scheme.

To see how the above observations depend on the ground motion characteristics, three accelerograms have been additionally considered. These are (i) the recorded S00E component at the El Centro site for the 1940 Imperial Valley earthquake, (ii) the recorded N85E component at the Cholame site for the 1966 Parkfield earthquake, and (iii) a synthetically generated motion at the Mexico City site for the 1985 Michoacan earthquake (see References 27 and 28 for details). It is assumed that these ground motions represent the 'expected' ground motions at the respective sites. The yield displacement levels for the EP oscillators for these cases have respectively been taken as 150, 100 and 500 mm. Although these choices of yield displacements may look somewhat arbitrary, the guided criterion behind these has been to limit the number of inelastic excursions to low levels assuming the oscillators to be mildly non-linear. Thus, in the case of Mexico City motion with a very large strong motion duration, a higher yield level had to be chosen, and in the case of Cholame motion with a short spurt of the energy release, a smaller value was found to be sufficient. Figures 3–5 respectively show the comparison of the spectra obtained from simulation with those

obtained via PSDFs for the linearized oscillators. It is seen that the above observations for the Loma Prieta earthquake hold good in these cases as well, and that the proposed linearized oscillator can indeed be used to approximate the EP oscillators.

The stochastic formulation for the inelastic spectra as proposed in the next section will be based on a conditional distribution involving the largest and lower-order displacement peaks. The linearization technique based on the equivalence of the largest maxima does not ensure strictly the equivalence of the lower-order peaks or the joint distribution of the largest and a lower-order peak. However, since the response of the linearized system may not be far away from the response of a mildly non-linear system, the proposed technique may be used to obtain a quick preliminary design estimate. Thus, the proposed linearized oscillator will be considered to obtain the inelastic damage spectra in this paper.

DAMAGE-DEPENDENT SPECTRAL ORDINATES

The inelastic spectra in this study are considered to consist of a set of curves representing the variation of linear design forces with equivalent time period, T_e, of the non-linear oscillator. Each curve in a set is obtained for a uniform value of maximum inelastic displacement, a (as normalized to the r.m.s. displacement), cumulative damage, d, damping ratio, ζ, or the duration, T, of the ground motion, with the remaining parameters held constant. The parameter d as considered in this study represents the cumulative damage normalized with respect to the total cumulative damage which occurs when all the displacement peaks exceed the yield displacement level.

The uniform damage inelastic spectra in this study are considered to consist of a set of curves, with each curve representing the variation of linear design forces with equivalent time period, T_e, of the non-linear oscillator for a uniform value of normalized cumulative damage. This normalization has been done with respect to the total cumulative damage corresponding to the situation of all displacement peaks exceeding the yield displacement level, i.e. $i = N$. A typical set of such curves corresponding to different damage levels is obtained for a particular value of the maximum inelastic displacement, a (as normalized to the r.m.s. displacement), damping ratio, ζ, and the duration, T, of the ground motion.

It has been illustrated in the previous section how to obtain the maximum inelastic displacement in a non-linear oscillator by using the equivalent linearization technique and the spectrum-compatible PSDF of the input excitation process. We can also estimate the yield displacement from the maximum displacement by using the 'expected' ductility ratio as formulated by Basu and Gupta[16] on the basis of the theory of order statistics. Let the maximum and yield displacements (as normalized to the r.m.s. displacement) be respectively identified as the first-order peak (largest maximum) and a higher-order peak (lower amplitude maximum) of the equivalent oscillator response. The ductility ratio, μ_i, thus becomes the ratio of the maximum level, a, to the yield level, b. Using the conditional distribution of the higher-order peaks, the expected value of this ratio is obtained as (see Reference 16 for details)

$$E(\mu_i) = \int_1^\infty \frac{(N-1)!}{(N-i-1)!\,(i-1)!}\,\frac{a}{\mu}\,\frac{[P(a/\mu)]^{N-i-1}[P(a) - P(a/\mu)]^{i-1}\,p(a/\mu)}{[P(a)]^{N-1}}\,\mathrm{d}\mu \tag{24}$$

where i is the number of inelastic excursions in the non-linear oscillator, and this is assumed to be the same as the number of excursions of the level b in the linearized oscillator. It may be noted that this formulation incorporates a statistical dependence between the maximum and the yield levels with i as a parameter. The parameter i plays an important role in the progressive damage of a structure, and estimation of this may depend on the cumulative damage we want to allow in the structure. Interpreting the cumulative damage in the normalized sense as mentioned before, Basu and Gupta[15] have observed a good correlation between this and the ratio i/N, irrespective of the values of N or b. Approximating this correlation to be linear in nature, the number of maximum allowable excursions, i, can be obtained as

$$i = dN \tag{25}$$

where d is the allowable damage in the normalized sense. We can consider this in an absolute sense also by multiplying with the cumulative damage for the N excursions, which has been shown by Basu and Gupta[15] to depend on the total number of peaks and the considered damage model.

Once the expected ductility ratio, $E(\mu_i)$, is estimated using this value of i in equation (24), the normalized yield displacement value, b, is obtained as

$$b = \frac{a}{E(\mu_i)} \tag{26}$$

On multiplying this now with the r.m.s. displacement of the equivalent oscillator, we obtain the yield displacement in the non-linear oscillator. This displacement may now be multiplied with ω_n^2 to estimate the design acceleration value, or, in other words, the desired spectral ordinate, SA, corresponding to T_e time period, ζ damping, and a (normalized) maximum displacement.

It follows from the above discussion that the inelastic spectra depend on several independent parameters, e.g. the allowable damage, d, the maximum inelastic displacement, a, the response duration, T, and the oscillator damping, ζ. To illustrate the effects of these parameters on the inelastic spectra, a filtered white noise model given by Kanai[29] and Tajimi[30] has been considered to characterize the input ground motion. The filter characteristics in this model have been taken as the ground natural frequency, $\omega_g = 2\pi$ rad/sec, and the ground damping ratio, $\zeta_g = 0{\cdot}2$, corresponding to the soft alluvium soil conditions, and the motion intensity corresponds to a r.m.s. ground acceleration of 0·1 g.

Figure 6 shows the inelastic spectra for damping ratio, $\zeta = 0{\cdot}05$, 0·1 and 0·2, with the other parameters taken as $a = 3{\cdot}0$, $d = 0{\cdot}1$ and $T = 40$ sec. These spectra show the variations of the spectral ordinates with the time period, T_e, of the equivalent oscillator. The spectral ordinates, SA, are multiplied with the ratio $(T_n/T_e)^2$ and then expressed in terms of g. The three curves for the different damping ratios are seen to follow the same trends as are generally observed for the elastic spectra. Further spectra in this paper will be obtained for the 5 per cent damping oscillators.

The inelastic spectra curves as in Figure 7 are obtained to study the effect of the variation of the response duration. For this, it has been assumed that $a = 3{\cdot}0$, $d = 0{\cdot}1$ and $T = 10$, 20 and 40 sec. Based on these results, the spectral values do not appear to be sensitive to the duration of the response process. However, since the longer durations are associated with greater absolute damages (see Reference 15), these spectral ordinates for any period actually correspond to different values of cumulative damage in the absolute sense. Those correspond to the same damage, as shown in the figure, only in the normalized sense.

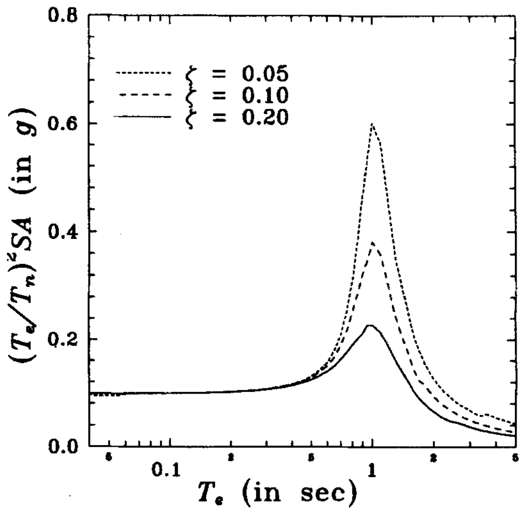

Figure 6. Inelastic spectra for different damping ratios with $a = 3{\cdot}0$, $d = 0{\cdot}1$ and $T = 40$ sec

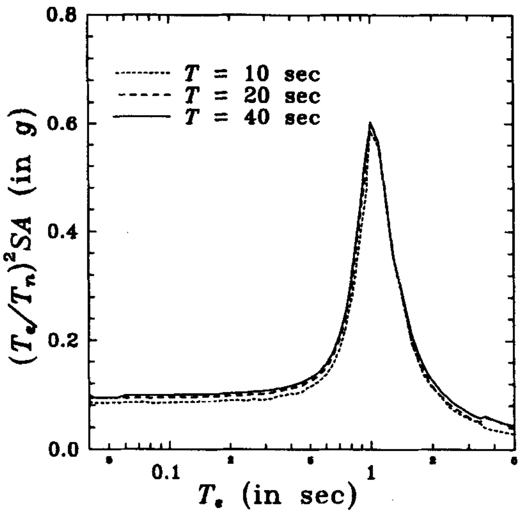

Figure 7. Inelastic spectra for different durations with $a = 3{\cdot}0$, $d = 0{\cdot}1$ and $\zeta = 0{\cdot}05$

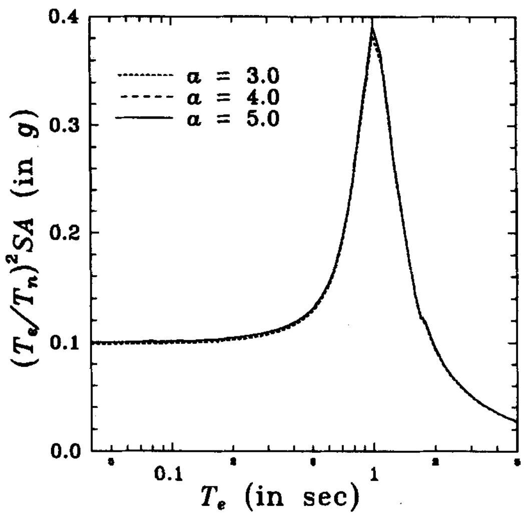

Figure 8. Inelastic spectra for different maximum displacements with $d = 0{\cdot}3$, $\zeta = 0{\cdot}05$ and $T = 40$ sec

Figure 8 represents the inelastic spectra for different values of normalized maximum displacement, a, with d and T respectively assumed to be 0·3 and 40 sec. It is observed that there is almost no effect of the variation in the normalized maximum level on the inelastic spectrum amplitudes. The associated invariance of the yield displacement, b, with the maximum displacement, a, for a given i and N is consistent with the observed linear variation of the expected ductility with a by Basu and Gupta.[16] This insensitivity of the spectrum amplitudes to the maximum inelastic displacement highlights the redundancy of limiting the maximum displacements in design, once there is a control on the cumulative damage. It is also clear that with the use of the proposed spectra, there remains no need to estimate the largest inelastic displacements by amplifying the linear displacements as is usually done through the empirical, period and site condition independent amplification factors.

The family of curves in Figure 9 are the spectra corresponding to different damage levels with a taken as 3·0 and T then as 10 sec. These uniform damage spectra clearly show that the structure has to be designed for greater forces if lower maximum damage is to be permitted.

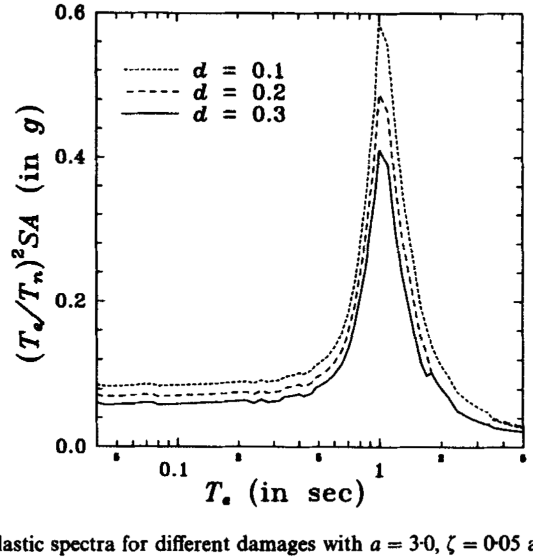

Figure 9. Inelastic spectra for different damages with $a = 3{\cdot}0$, $\zeta = 0{\cdot}05$ and $T = 10$ sec

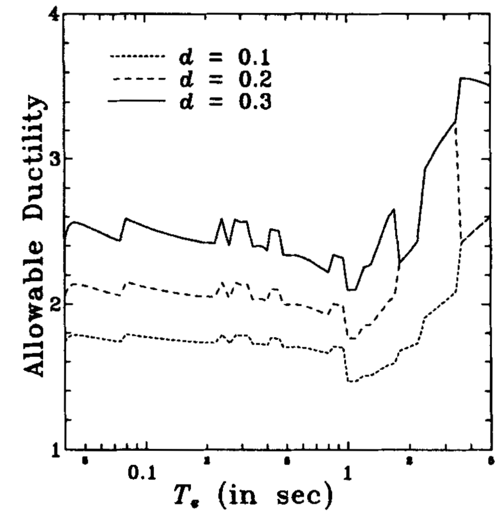

Figure 10. Allowable ductility curves for different damages with $a = 3{\cdot}0$ and $T = 10$ sec in the case of soft alluvium sites

The curves in Figure 10 show the variation of expected ductility as computed by using equation (24) with the period, T_e, of the equivalent oscillators for different damage levels, with a taken as 3·0 and T taken as 10 sec. These curves are useful in estimating the maximum allowable ductility demand for the damage to remain within the specified limit. In general, the curves show that for more allowable damage, the allowable ductility is more. This is consistent with the earlier observations of Basu and Gupta.[15] Further, the curve for $d = 0{\cdot}2$ shows abrupt jumps for the longer period oscillators following the number of peaks, N, becoming very small for the considered duration of 10 sec. In this situation, whether $d = 0{\cdot}1$ and 0·2 or $d = 0{\cdot}2$ and 0·3, both cases correspond to the same number of excursions as obtained from equation (25), and thus to the same ductility. To see the effect of site conditions on these curves, another case has been considered with $\omega_g = 5\pi$ and $\zeta_g = 0{\cdot}6$ corresponding to the hard rocky site. Figure 11 shows the variation of allowable ductility with T_e for this case. On comparison with the results in Figure 10, it is observed that the allowable ductility may be more for the same damage in the case of structures in the medium period range when those are sitting on

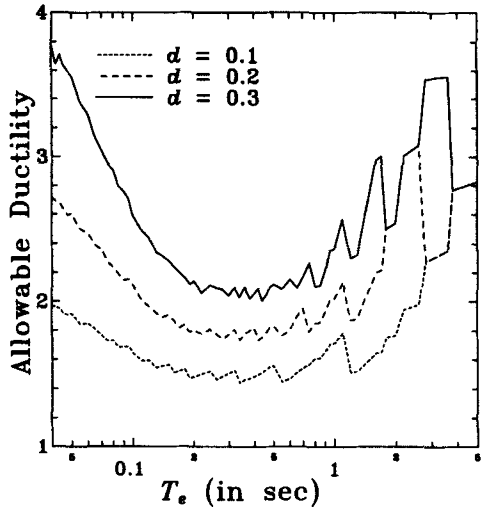

Figure 11. Allowable ductility curves for different damages with $a = 3{\cdot}0$ and $T = 10$ sec in the case of hard rock sites

the soft alluvium as compared to the hard rock site. For the flexible structures, however, the allowable ductility seems to depend little on the site conditions. The observed jumps here in the long period range are also due to the lower value of N as in Figure 10.

CONCLUSIONS

A stochastic approach has been proposed here for the computation of damage-dependent inelastic spectra in the case of SDOF non-linear oscillators idealized as elasto-plastic in nature. These oscillators have been linearized through a new scheme which is based on the equivalence of the largest maximum response rather than on the mean square response. Through numerical simulation results, it has been shown that the proposed formulation gives reasonably accurate and slightly better results of the maximum inelastic displacement compared to those based on the traditional criterion.

A parametric study on the proposed inelastic spectra has shown that restricting the normalized cumulative damage due to the inelastic excursions reduces the sensitivity of the spectral amplitudes to the maximum inelastic response and to the response duration. It is also observed that the allowable ductility increases if more damage is allowed, or when the structure is neither stiff nor flexible and is situated on a soft alluvium soil.

The proposed approach to the inelastic spectra can be extended easily to other types of non-linear oscillators, including the hysteretic oscillators, through the formulation of appropriate equivalent oscillator properties. The spectra presented in the paper form an essential part of a new damage-based design philosophy. This philosophy integrates the concepts behind the traditional strength and displacement spectra through control on a more physical parameter, i.e. the cumulative damage.

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